CHAPTER 2

Motion in a Straight Line

Topic Covered

- Mechanics
- Position
- Types of Motion
- Distance & Displacement
- Speed & Velocity
- Instantaneous velocity and speed
- Acceleration
- Position-Time Graph
- Velocity-Time Graph
- Kinematic equations for uniformly accelerated motion
- Free fall
- Motion with variable acceleration

INTRODUCTION

MECHANICS

The branch of physics which deals with the study of motion of the objects is known as mechanics.

Three branches of mechanics:

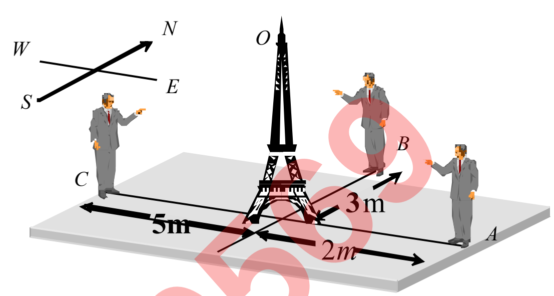
(i) Statics: The branch of physics which deals with the study of motion of the objects or bodies in equilibrium under the action of external forces.

(ii) Kinematics: The branch of physics which deals with the study of mechanical motion without taking into account the cause of the motion in the bodies.

(iii) Dynamics: The branch of physics which deals with the study of mechanical motion by taking into the account the cause/forces of the motion in the bodies.

POSITION

An object situated at point O is observed by three observers from three different places, all three observers have different observations about the position of point O and no one is wrong. Because they are observing the object from their different positions.



Observer 'A' says : Point O is 2 m away in west direction.

Observer 'B' says : Point O is 3 m away in south direction. Observer 'C' says : Point O is 5 m away in east direction.

Therefore position of any point is completely expressed by two factors: Its distance from the observer and its direction with respect to the observer.

That is why position is characterized by a vector known as position vector.

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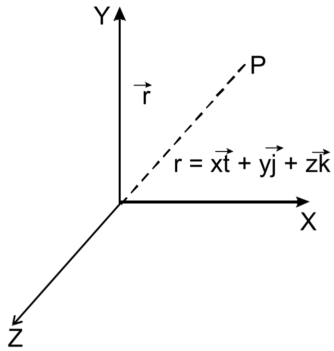


Let point P is in a xy plane and its coordinates are (x, y). Then position vector $(\vec{r})$ of point will be $x\hat{i} + y\hat{j}$ and if the point P is in a space and its coordinates are (x, y, z) then position vector can be expressed as $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

Rest : An object is said to be at rest if it does not change its position with time as wellas with respect to its surroudings. A book lying on a table, a person sitting in a chair are the examples of rest.

Motion : An object is said to be in motion if it changes its position with time as well as with respect to its surroundings.

Example : A bird flying in air, a train moving on rails, a ship sailing on water, a man walking on road are some of the examples of motion, visible to the eye. Movement of gas molecules is also an example of motion, which is invisible to the eye.

Rest & Motion are relative terms : When we say that an object is at rest or in motion,then this statement is incomplete and meaningless. Basically, rest & motion are relative terms. An object which is at rest can also be in motion simultaneously. This can be illustrated as follows.

FRAME OF REFERENCE

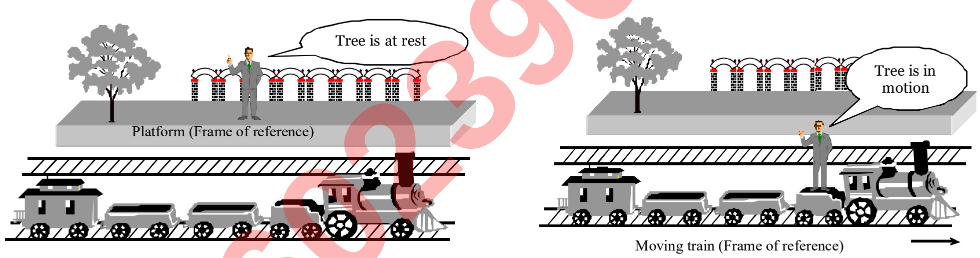
Frame of Reference: It is a system to which a set of coordinates are attached and with reference to which observer describes any event. Position of an object is specified with respect to a **reference frame.**

In a reference frame, an observer measures the position of the other object at any instant of time, with respect to a coordinate system chosen and fixed arbitrary on the reference frame.

If a body does not change its position as time passes with respect to frame of reference, it is said to be at rest.

It is said to be in motion if it changes its position as time passes with respect to frame of reference. A passenger standing on the platform observes that tree on a platform is at rest, but when the same passenger is passing away in a train through station, observes that tree is in motion. In both conditions observer is right. But the observations are different because in the first situation observer stands on the platform, in which reference frame is at rest and in second situation observer is moving in train, that is reference frame in motion.

So rest and motion are relative terms. It depends upon the frame of references



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TYPES OF MOTION.

One dimensional

Motion of a body in a straight line is called one dimensional motion or rectilinear motion. When only one coordinate of the position of a body changes with time then it is said to be moving one dimensionally.

e.g.. Motion of ant on a straight line and motion of freely falling body.

Two dimensional

Motion of body in a plane is called two dimensional motion.

When two coordinates of the position of a body changes with time then it is said to be moving two dimensionally.

e.g. Motion of van on a circular turn and motion of billiards ball.

Three Dimensional

Motion of body in a space is called three dimensional motion.When all three coordinates of the position of a body changes with time then it is said to be moving three dimensionally.e.g. Motion of flying kite and motion of flying insect in air.

PARTICLE OR POINT MASS

The smallest part of matter with zero dimensions which can be described by its mass and position is defined as a particle.

If the size of a body is negligible in comparison to its range of motion then that body is known as a point object.

A body (Group of particles) to be known as a particle depends upon types of motion. For example in a planetary motion around the sun the different planets can be presumed to be the particles.

In above consideration when we treat body as particle, all parts of the body undergo same displacement and have same velocity and acceleration.

DISTANCE AND DISPLACEMENT

Distance: It is the actual path length covered by a moving particle in a given interval of time.

Special note:

1. The actual disatnce travelled by a particle in the given interval of time is always equal to or greater than the magnitude of the displacement and in no case, it is less than the magnitude of the displacement,

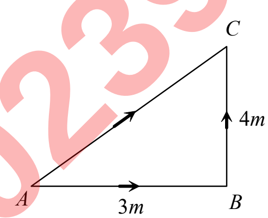
i.e., Distance $\geq$ | Displacement |

2. Displacement may be + ve, – ve or zero.

3. Distance, speed and time can never be negative.

4. At the same time particle cannot have two positions.

Distance = 3 m + 4 m = 7 m



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Displacement : Displacement is the change in position vector i.e., A vector joining initial to final position. Its magnitude is the shortest distance between the initial and the final position of the object.

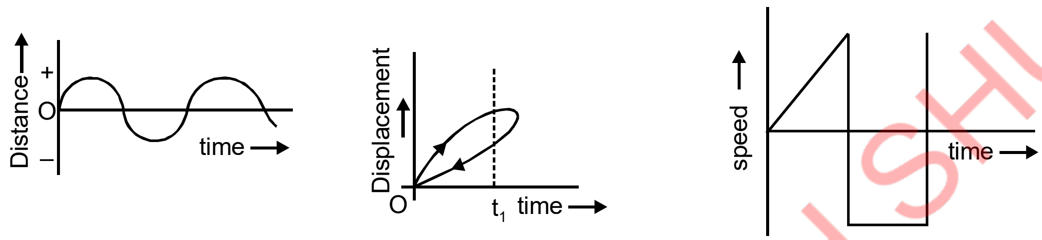
1. Displacement is a vector quantity

2. Dimension : $[M^0L^1T^0]$

3. Unit : metre (S.I.)

4. If $\vec{S}_1, \vec{S}_2, \vec{S}_3........\vec{S}_n$ are the displacements of a body then the total (net) displacement is the vector sum of the individuals. $\vec{S} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3 + ... + \vec{S}_n$

Some Impossible Graphs



SOLVED EXAMPLE

Example 1. An old person moves on a semi circular track of radius 40 m during a morning walk. If he starts at one end of the track and reaches at the other end. Find the displacement of the person.

Solution. Displacement = 2R = 2 x 40 = 80 meter.

Example 2. An athelete is running on a circular track of radius 50 meter. Calculate the displacement of the athlete after completing 5 rounds of the track.

Solution. Since final and initial positions are same .

Hence displacement of athlete will be $\Delta r = r - r = 0$

Example 3. If a particle moves from point A to B then distance covered by particle will be.

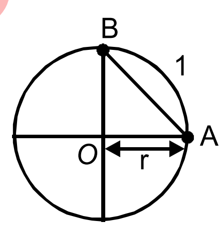
Solution. D = x + 2x = 3x

Example 4. A body covers $\frac{1}{4}$ th part of a circular path. Calulate the ratio of distance and displacement.

Solution. Distance = AB from path (1) Displacement = AB

$$= \frac{2\pi r}{4} = \frac{\pi r}{2} \qquad = \sqrt{OA^2 + OB^2}$$

$$= \sqrt{r^2 + r^2} = r\sqrt{2}$$



$$\therefore \quad \frac{\text{Distance}}{\text{Displacement}} = \frac{\pi r/2}{r\sqrt{2}} = \frac{\pi}{2\sqrt{2}}$$

Example 5. A point P consider at contact point of a wheel on ground which rolls on ground without sliping then value of displacement of point P when wheel completes half of rotation - [If radius of wheel is 1 m]

Solution. Displacement = $\sqrt{\pi^2 + 4}$ m

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EXERCISE

1. A man goes 10m towards North, then 20m towards east then displacement is

(A) 22.5m (B) 25m

(C) 25.5m (D) 30m

2. A body moves over one fourth of a circular arc in a circle of radius r. The magnitude of distance travelled and displacement will be respectively

(A) $\frac{\pi r}{2}, r\sqrt{2}$ (B) $\frac{\pi r}{4}, r$

(C) $\pi r \frac{r}{\sqrt{2}}$ (D) πr, r

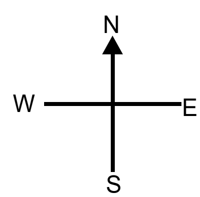
3. The displacement of the point of the wheel initially in contact with the ground, when the wheel roles forward half a revolution will be (radius of the wheel is R)

(A) $\frac{R}{\sqrt{\pi^2+4}}$ (B) $R\sqrt{\pi^2+4}$

(C) $2\pi R$ (D) πR

4. A monkey is moving on circular path of radius 80 m. Calcualte the distance covered by the monkey.

5. A man has to go 50 m due north, 40 m due east and 20 m due south to reach a field.



(A) What distance he has to walk to reach the field?

(B) What is his displacement from his house to the field?

SPEED AND VELOCITY

Speed : Rate of distance covered with time is called speed.

$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

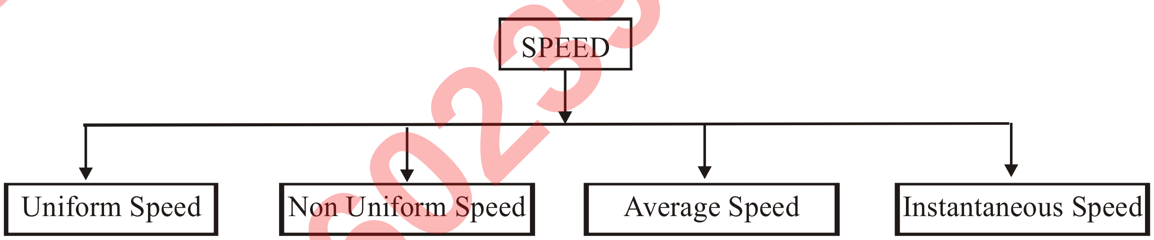
(i) It is a scalar quantity having symbol υ .

(ii) Dimension : $[M^0L^1T^{-1}]$

(iii) Unit : metre/second (S.I.), cm/second (C.G.S.)

$1 km/h = \frac{100}{60 \times 60} = \frac{5}{18} m/s$ $\quad 1 km/h = \frac{5}{18} m/s$

(iv) Types of speed :



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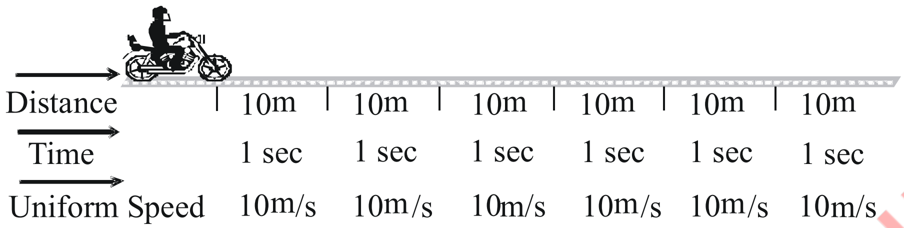
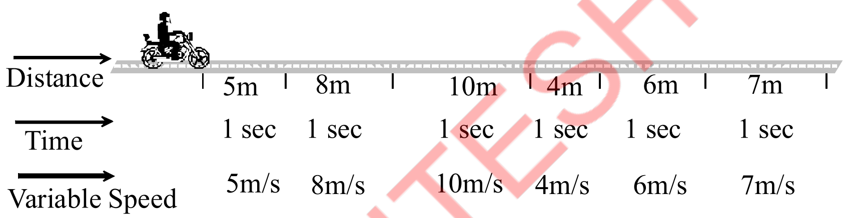
(A) Uniform speed : When a particle covers equal distances in equal intervals of time, (no matter how small the intervals are) then it is said to be moving with uniform speed. In the given example motorcyclist travels equal distance (= 10m) in each second. So we can say that particle is moving with uniform speed of 10 m/s.



(B) Non-uniform (variable) speed : If a body covers unequal distances in equal intervals of time it is said to be in non-uniform speed . In the given example motorcyclist travels 5m in 1^{st} second, 8m in 2^{nd} second, 10m in 3^{rd} second, 4m in 4^{th} second etc.

Therefore its speed is different for every time interval of one second. This means particle is moving with variable speed.



(C) Average speed : The average speed of a particle for a given 'Interval of time' is defined as the ratio of distance traveled to the time taken.

$$\text{Average speed} = \frac{\text{Distance travelled}}{\text{Time taken}}; \quad v_{av} = \frac{\Delta s}{\Delta t}$$

Time average speed : When particle moves with different uniform speed υ_1, υ_2, υ_3 ... etc in different time intervals t_1, t_2, t_3, ... etc respectively, its average speed over the total time of journey is given as

$$v_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}}$$

$$= \frac{d_1 + d_2 + d_3 + \ldots}{t_1 + t_2 + t_3 + \ldots.} = \frac{v_1 t_1 + v_2 t_2 + v_3 t_3 + \ldots}{t_1 + t_2 + t_3 + \ldots}$$

Special case : When particle moves with speed v_1 up to half time of its total motion and in rest time it is moving with speed v_2 then $v_{av} = \frac{v_1 + v_2}{2}$

Distance averaged speed : When a particle describes different distances d_1, d_2, d_3,...... with different time intervals t_1, t_2, t_3 ...... with speeds v_1, v_2, v_3...... respectively then the speed of particle averaged over the total distance can be given as

$$\upsilon_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{d_1 + d_2 + d_3 + \ldots\ldots}{t_1 + t_2 + t_3 + \ldots\ldots} = \frac{d_1 + d_2 + d_3 + \ldots\ldots}{\frac{d_1}{\upsilon_1} + \frac{d_2}{\upsilon_2} + \frac{d_3}{\upsilon_3} + \ldots\ldots}$$

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(1) When particle moves the first half of a distance at a speed of v_1 and second half of the distance at speed v_2 then $v_{av} = \dfrac{2v_1v_2}{v_1 + v_2}$

(2) When particle covers one-third distance at speed v_1, next one third at speed v_2 and last one third at speed v_3, then $v_{av} = \dfrac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$

Instantaneous speed: It is the speed of a particle at particular instant. When we say "speed", it usually means instantaneous speed.

The instantaneous speed is average speed for infinitesimally small time interval (i.e.,). Thus Instantaneous speed $v = \lim\limits_{\Delta t \to 0} \dfrac{\Delta s}{\Delta t} = \dfrac{ds}{dt}$

Velocity : Rate of change of position i.e. rate of displacement with time is called velocity.

(i) It is a scalar quantity having symbol v .

(ii) Dimension : $[M^0L^1T^{-1}]$

(iii) Unit : metre/second (S.I.), cm/second (C.G.S.)

TYPES OF VELOCITY

(A) Uniform velocity : A particle is said to have uniform velocity, if magnitudes as well as direction of its velocity remains same and this is possible only when the particles moves in same straight line without reversing its direction.

(B) Non-uniform velocity : A particle is said to have non-uniform velocity, if either of magnitude or direction of velocity changes (or both changes).

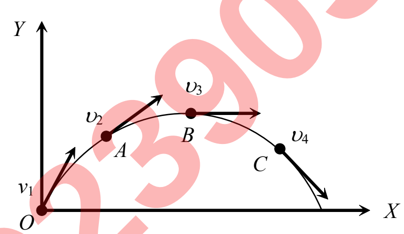
(C) Average velocity : It is defined as the ratio of displacement to time taken by the body ;

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time taken}}\ ; \quad \vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t}$$

(D) Instantaneous velocity: Instantaneous velocity is defined as rate of change of position vector of particles with time at a certain instant of time or the velocity at a particular moment of time is known as instantaneous velocity. The term "velocity" usually means instantaneous velocity.

$$\text{Instantaneous velocity } \vec{v} = \lim_{t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

COMPARISON BETWEEN INSTANTANEOUS SPEED AND INSTANTANEOUS VELOCITY



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(A) instantaneous velocity is always tangential to the path followed by the particle. When a stone is thrown from point O then at point of projection the instantaneous velocity of stone is v_1, at point A the instantaneous velocity of stone is v_2, similarly at point B and C are v_3 and v_4 respectively.

Direction of these velocities can be found out by drawing a tangent on the trajectory at a given point.

(B) A particle may have constant instantaneous speed but variable instantaneous velocity.

Example : When a particle is performing uniform circular motion then for every instant of its circular motion its speed remains constant but velocity changes at every instant.

(C) The magnitude of instantaneous velocity is equal to the instantaneous speed.

(D) If a particle is moving with constant velocity then its average velocity and instantaneous velocity are always equal.

(E) If displacement is given as a function of time, then time derivative of displacement will give velocity.

Let displacement $x = A_0 - A_1 t + A_2 t^2$

Instantaneous velocity $v = \frac{d\vec{x}}{dt} = \frac{d}{dt}(A_0 - A_1 t + A_2 t^2)$

$\vec{v} = -A_1 + 2A_2 t$

For the given value of t, we can find out the instantaneous velocity.

e.g. for t = 0,Instantaneous velocity $\vec{v} = -A_1$ and Instantaneous speed $|\vec{v}| = A_1$

COMPARISON BETWEEN AVERAGE SPEED AND AVERAGE VELOCITY

(A) Average speed is scalar while average velocity is a vector both having same units (m/s) and dimensions $[LT^{-1}]$.

(B) Average speed or velocity depends on time interval over which it is defined.

(C) For a given time interval average velocity is single valued while average speed can have many values depending on path followed.

(D) If after motion body comes back to its initial position then $\vec{v}_{av} = \vec{0}$ (as $\Delta\vec{r} = 0$) but $v_{av} > \vec{0}$ and finite as $(\Delta s > 0)$.

(E) For a moving body average speed can never be negative or zero (unless $t \to \infty$) while average velocity can be i.e. negative, zero or positve..

e.g. for t = 0,Instantaneous velocity $\vec{v} = -A_1$ and Instantaneous speed $|\vec{v}| = A_1$

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SOLVED EXAMPLE

Example 6. A man walks at a speed of 6 km/hr for 1 km and 8 km/hr for the next 1 km. What is his average speed for the walk of 2 km.

Solution. $\bar{V}=\frac{2v_1v_2}{v_1+v_2}=\frac{2\times6\times8}{6+8}=7\,km/h$

Example 7. The distance travelled by a particle $S = 10t^2$ (m). Find the value of instantaneous speed at t = 2 sec.

Solution. $V=\frac{dx}{dt}=\frac{d}{dt}\left(10t^2\right)=10(2t)=20t$

Put t = 2 sec. V = 20 × 2 = 40 m/s

Example 8. A car travels a distance A to B at a speed of 40 km/h and returns to A at a speed of 30 km/h.

(i) What is the average speed for the whole journey?

(ii) What is the average velocity ?

Solution. (i) Let AB = s, time taken to go from A to B, $t_1 = \frac{s}{40}h$

and time taken to go from B to A, $t_2 = \frac{s}{30}h$

$\therefore$ total time taken = $t_1 + t_2 = \frac{s}{40}+\frac{s}{30}=\frac{(3+4)s}{120}=\frac{7s}{120}h$

Total distance travelled = s + s = 2s

$$\text{Average speed}=\frac{\text{total distance travelled}}{\text{total time taken}}=\frac{2s}{7s/120}=\frac{120\times2}{7}=34.3\,km/h$$

(ii) Total displacement = zero, since the car returns to the original position.

$$\text{Therefore, average velocity}=\frac{\text{total displacement}}{\text{time taken}}=\frac{0}{2t}=0$$

Example 9. From the adjoining position time graph for two particles A and B the ratio of velocities $v_A : v_B$ will be

(A) 1 : 2 (B) $1:\sqrt{3}$

(C) $\sqrt{3}:1$ (D) 1 : 3

Solution. $\frac{V_A}{V_B}=\frac{\tan\theta_a}{\tan\theta_b}=\frac{\tan30^0}{\tan60^0}=\frac{1/\sqrt{3}}{\sqrt{3}}=\frac{1}{3}$

Example 10. The position of a particle moving on x-axis is given by $x = At^3 + Bt^2 + Ct + D$. The numerical value of A, B, C, D are 1, 4, –2 and 5 respectivley and S.I. units are used. Find velocity of the particle at t = 4 sec.

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Solution. $V = \frac{dx}{dt} = \frac{d}{dt}$ $[At^3 + Bt^2 + Ct + D]$

or $V = 3At^2 + 2Bt + C$

at time t = 4 sec. Considering A = 1, B = 4, C = –2

$V = 3A\ (4)^2 + 2B\ (4) + C$

$V = 48\ (0) + 8\ (4) + (-2)$

$V = 48\ A + 8B + C = 78$ m/s

Example 11. In a car race, car A takes a time of t sec. less than car B at the finish and passes the finishing point with a velocity υ m/s more than the car B. Assuming that the cars start from rest and travel with constant accelerations a_1 and a_2 respectively, show that $v = \left[\sqrt{a_1 a_2}\right] t$

Solution. Let the time taken by two cars to complete the journey be t_1 and t_2 and their velocities at the finish be υ_1 and υ_2 respectively.

Given that, $t_1 = t_2 - t$ and $\upsilon_1 = \upsilon_2 + \upsilon$ ... (1)

Now $s_1 = \frac{1}{2} a_1 t_1^2$ and $S_2 = S = \frac{1}{2} a_2 t_2^2$ ... (2)

(At start, $v_1 = v_2 = 0$) ... (3)

Hence $a_1 t_1^2 = a_2 t_2^2 = 2s$

Also, $v_1 = a_1 t_1$ and $v_2 = a_2 t_2$

or $v_1 t_1 = a_1 t_1^2 = 2s$ and $v_2 t_2 = a_2 t_2^2 = 2s$

$\therefore \quad t_1 = \frac{2s}{v_1}$ and $t_2 = \frac{2s}{v_2}$

So, $t_2 - t_1 = 2s\left[\frac{1}{v_2} - \frac{1}{v_1}\right]$ . .... (4)

From equations (1) and (4) we have

$$2s\left[\frac{1}{v_2} - \frac{1}{v_1}\right] = t$$

or $2s\left[\frac{v_1 - v_2}{v_1 v_2}\right] = t$ or $2s\left[\frac{v}{v_1 v_2}\right] = t$

or $v = \left[\frac{v_1 v_2}{2s}\right] t = \sqrt{\left\{\frac{v_1^2 v_2^2}{(2s)^2}\right\}} \times t$

$$\sqrt{\left(\frac{v_1 v_2}{t_1 t_2}\right)} \times t = \sqrt{(a_1 a_2)} \times t$$

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EXERCISE

6. If a car covers 2/5th of the total distance with v_1 speed and 3/5th distance with v_2 then average speed is

(A) $\frac{1}{2}\sqrt{v_1 v_2}$ (B) $\frac{v_1+v_2}{2}$

(C) $\frac{2v_1+v_2}{v_1+v_2}$ (D) $\frac{5v_1+v_2}{3v_1+2v_2}$

7. A car accelerated from initial position and then returned at initial point, then

(A) Velocity is zero but speed increases (B) Speed is zero but velocity increases

(C) Both speed and velocity increase (D) Both speed and velocity decrease

8. A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km/h. Finding the market closed, he instantly turns and walks back home with a speed of 7.5 km/h. The average speed of the man over the interval of time 0 to 40 min. is equal to

(A) 5 km/h (B) $\frac{25}{4}$ km/h

(C) $\frac{30}{4}$ km/h (D) $\frac{45}{8}$ km/h

9. The relation $3t = \sqrt{3x} + 6$ describes the displacement of a particle in one direction where x is in metres and t in sec. The displacement, when velocity is zero, is

(A) 24 metres (B) 12 metres

(C) 5 metres (D) Zero

10. The motion of a particle is described by the equation $x = a + bt^2$ where a = 15cm and b = 3cm. Its instantaneous velocity at time 3 sec will be

(A) 36 cm/sec (B) 18 cm/sec

(C) 16 cm/sec (D) 32 cm/sec

11. A train has a speed of 60 km/h for the first one hour and 40 km/h for the next half hour. Its average speed in km/h is

(A) 50 (B) 53.33

(C) 48 (D) 70

12. A person completes half of its his journey with speed υ_1 and rest half with speed υ_2. The average speed of the person is

(A) $v = \frac{v_1+v_2}{2}$ (B) $v = \frac{2v_1+v_2}{v_1+v_2}$

(C) $v = \frac{v_1+v_2}{v_1+v_2}$ (D) $v = \sqrt{v_1 v_2}$

13. A car moving on a straight road covers one third of the distance with 20 km/hr and the rest with 60 km/hr. The average speed is

(A) 40 km/hr (B) 80 km/hr

(C) $46\frac{2}{3}$ km/hr (D) 36 km/hr

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ACCELERATION

The time rate of change of velocity of an object is called acceleration of the object.

(1) It is a vector quantity. It's direction is same as that of change in velocity (Not of the velocity)

(2) There are three possible ways by which change in velocity may occur

When only direction of velocity changes	**When only magnitude of velocity changes**	**When both magnitude and direction of velocity changes**
Acceleration perpendicular to velocity	Acceleration parallel or anti-parallel to velocity	Acceleration has two components one is perpendicular to velocity and another parallel or anti-parallel to velocity
e.g. Uniform circular motion	*e.g.* Motion under gravity	*e.g.* Projectile motion

(3) Dimension : $[M^0L^1T^{-2}]$

(4) Unit : metre/second^2 (S.I.); cm/second^2 (C.G.S.)

(5) Types of acceleration :

i. **Uniform acceleration :** A body is said to have uniform acceleration if magnitude and direction of the acceleration remains constant during particle motion.

Note : If a particle is moving with uniform acceleration, this does not necessarily imply that particle is moving in straight line. e.g. Projectile motion.

ii. **Non-uniform acceleration :** A body is said to have non-uniform acceleration, if magnitude or direction or both, change during motion.

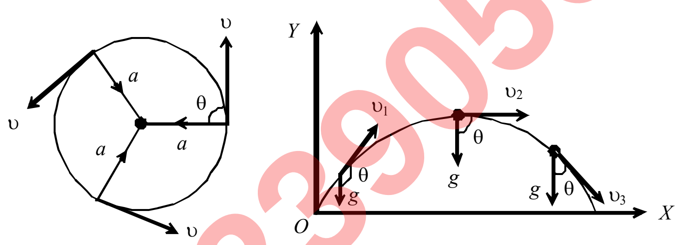
iii. **Average acceleration :** $\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta\vec{t}} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$

The direction of average acceleration vector is the direction of the change in velocity vector as

$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$

iv. **Instantaneous acceleration** = $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$

v. For a moving body there is no relation between the direction of instantaneous velocity and direction of acceleration.



e.g. (A) In uniform circular motion θ = 90° always

(B) In a projectile motion θ is variable for every point of trajectory.

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vi. If a force $\vec{F}$ acts on a particle of mass m, by Newton's 2^{nd} law, acceleration $\vec{a} = \frac{\vec{F}}{m}$

vii. By definition $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}\left[\text{As } \vec{v} = \frac{d\vec{x}}{dt}\right]$

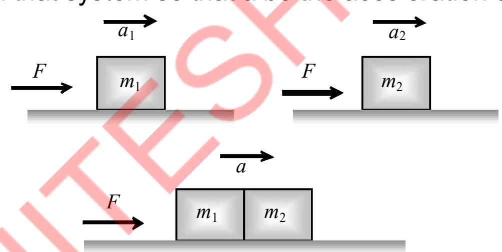
i.e., if x is given as a function of time, second time derivative of displacement gives acceleration

viii. If velocity is given as a function of position, then by chain rule

$$a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = v.\frac{dv}{dx}\left[\text{as } v = \frac{dx}{dt}\right]$$

ix. If a particle is accelerated for a time t_1 by acceleration a_1 and for time t_2 by acceleration a_2 then average acceleration is $a_{av} = \frac{a_1t_1 + a_2t_2}{t_1 + t_2}$

x. If same force is applied on two bodies of different masses m_1 and m_2 separately then it produces accelerations a_1 and a_2 respectively. Now these bodies are attached together and form a combined system and same force is applied on that system so that a be the acceleration of the combined system, then



$$F = (m_1 + m_2) \Rightarrow \frac{F}{a} = \frac{F}{a_1} + \frac{F}{a_2}$$

$$\text{So, } \frac{1}{a} = \frac{1}{a_2} + \frac{1}{a_2} \Rightarrow a = \frac{a_1a_2}{a_1 + a_2}$$

xi. Acceleration can be positive, zero or negative. Positive acceleration means velocity increasing with time, zero acceleration means velocity is uniform constant while negative acceleration (retardation) means velocity is decreasing with time.

xii. For motion of a body under gravity, acceleration will be equal to "g", where g is the acceleration due to gravity. Its normal value is 9.8 m/s^2 or 980 cm/s^2 or 32 $feet/s^2$..

Special Note - I

If the motion of a particle is accelerated translatory (without change in direction) $\vec{v} = |\vec{v}|\hat{n}$

$$\Rightarrow \frac{d\vec{v}}{dt} = \frac{d}{dt}[|\vec{v}|\hat{n}] = \hat{n} = \hat{n}\frac{d|\vec{v}|}{dt} \quad [\text{as } \vec{n} \text{ is constt.}] \Rightarrow \left|\frac{d\vec{v}}{dt}\right| = \frac{d\vec{v}}{dt}(\neq 0)$$

Howeverm, if motion is uniform translatory, both these will still be equal but zero.

Special Note - II

(i) $\frac{d|\vec{v}|}{dt} = 0$ while $\left|\frac{d\vec{v}}{dt}\right| \neq 0$ (it is possible.)

(ii) $\frac{d|\vec{v}|}{dt} \neq 0$ while $\left|\frac{d\vec{v}}{dt}\right| = 0$ (it is not possible.)

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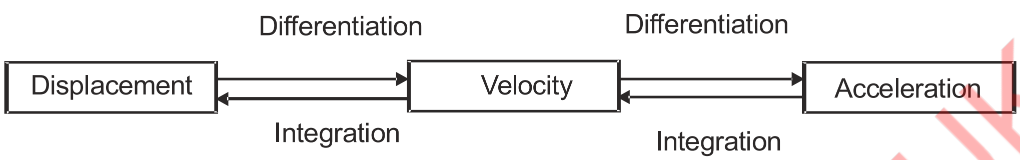
EFFECTIVE USE OF MATHEMATICAL TOOLS IN SOLVING PROBLEMS OF ONE-DIMENSIONAL MOTION

If displacement-time equation is given, we can get velocity-time equation with the help of differentiation. Again, we can get acceleration-time equation with the help of differentiation.

If acceleration-time equation is given, we can get velocity-time equation by integration. From velocity equation, we can get displacement-time equation by integration.



SOLVED EXAMPLE

Example 12. The displacement of a particle is given by $y = a + bt + ct^2 + dt^4$. Find the acceleration of a particle.

Solution. $= \frac{dy}{dt} = \frac{d}{dt}\left(a + bt + ct^2 + dt^4\right) = b + 2ct + 4dt^3$

$$a = \frac{dv}{dt} = 2c + 12dt^2$$

Example 13. If the displacement of a particle is $(2t^2 + t + 5)$ meter then, what will be acceleration at t = 5 sec.

Solution. $v = \frac{dx}{dt} = \frac{d}{dt}\left(2t^2 + t + 5\right) = 4t + 1\ m/s$ and $a = \frac{dv}{dt} = \frac{d}{dt}\left(4t + 1\right)$ $a = 4\ m/s$

Example 14. The velocity of a particle moving in the x direction varies as $V = \alpha\sqrt{x}$ where α is a constant. Assuming that at the moment t = 0 the particle was located at the point x = 0. Find the acceleration.

Solution. $a = \frac{dv}{dt} = \alpha\frac{d}{dt}\sqrt{x} = \alpha.\frac{1}{2}x^{-1/2}.\frac{dx}{dt}$

$$= \alpha.\frac{1}{2\sqrt{x}}.\alpha\sqrt{x} \quad \Rightarrow \quad a = \frac{\alpha^2}{2}$$

Example 15. The velocity of any particle is related with its displacement As; $x = \sqrt{v + 1}$, Calculate acceleration at x = 5 m.

Solution. $x = \sqrt{v + 1}$ $\quad x^2 = v + 1$ $\quad v = \left(x^2 - 1\right)$

Therefore $a = \frac{dv}{dt} = \frac{d}{dt}(x^2 - 1) = 2x\frac{dx}{dt} = 2x\ v = 2x(x^2 - 1)$

at x = 5 m $a = 2 \times 5\left(25 - 1\right) = 240 m/s^2$

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Example 16. The velocity of a particle moving in the positive direction of x-axis varies as v = $\alpha\sqrt{x}$ where α is positive constant. Assuming that at the moment t = 0, the particle was located at x = 0 find, (i) the time dependance of the velocity and the acceleration of the partic le and (ii) the mean velocity of the particle averaged over the time that the particle takes to cover first s metres of the path.

Solution. **(i)** Given that $v = \alpha\sqrt{x}$ or $\frac{dx}{dt} = \alpha\sqrt{x}$

$$\therefore \quad \frac{dx}{\sqrt{x}} = \alpha\,dt \quad \text{or} \quad \int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \alpha\,dt$$

Hence $2\sqrt{x} = \alpha\,t \text{ or } x = (\alpha^2 t^2 / 4)$

Velocity $\frac{dx}{dt} = \frac{1}{2}\alpha^2 t$ and

Acceleration $\frac{d^2x}{dt^2} = \frac{1}{2}\alpha^2$

(ii) Time taken to cover first s metres

$$s = \frac{\alpha^2 t^2}{4} \quad \text{or} \quad t^2 = \frac{4s}{\alpha^2} \quad \text{or} \quad t = \frac{2\sqrt{s}}{\alpha}$$

$$\bar{v} = \frac{\text{total distance}}{\text{total time}} = \frac{s\alpha}{2\sqrt{s}} \text{ or } \bar{v} = \frac{1}{2}\sqrt{s}\,\alpha$$

Example 17. A particle moves in the plane xy with constant acceleration a directed along the negative y-axis. The equation of motion of the particle has the form y = px – qx^2 where p and q are positive constants. Find the velocity of the partcle at the origin of coordinates.

Solution. Given that y = px – qx^2

$$\therefore \quad \frac{dy}{dx} = p\frac{dx}{dt} - q.2x\frac{dx}{dt} \quad \text{and} \quad \frac{d^2y}{dt^2} = p\frac{d^2x}{dt^2} - 2qx\frac{d^2x}{dt^2} - 2q\left(\frac{dx}{dt}\right)^2$$

$$\text{or} \quad -a = -2q\left(\frac{dx}{dt}\right)^2 = -2q\upsilon_x^2$$

$$\because \quad \frac{d^2x}{dt^2} = 0 \text{ (no acceleration along x-axis) and } \frac{d^2y}{dt^2} = -a$$

$$\therefore \quad \upsilon_x^2 = \frac{a}{2q} \quad \text{or} \quad \upsilon_x = \sqrt{\frac{a}{2q}}$$

Further, $\left(\frac{dy}{dt}\right)_{x=0} = p\frac{dx}{dt}$

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or $\upsilon_y = p\upsilon_x$

$$\therefore \qquad \upsilon_y = p\sqrt{\left(\frac{a}{2q}\right)}$$

Now $\upsilon = \sqrt{\left(\upsilon_x^2 + \upsilon_y^2\right)} = \sqrt{\left(\frac{a}{2q} + \frac{ap^2}{2q}\right)}$

or $$\upsilon = \sqrt{\left[\frac{a\left(p^2+1\right)}{2q}\right]}$$

Example 18. A particle is moving in a plane with velocity given by $u = u_0 i + (a\omega\cos\omega t)j$, where i and j are unit vectors along x and y axes respectively. If particle is at the origin at t = 0.

(A) Calculate the trajectory of the particle.

(B) Find the distance from the origin at time $3\pi/2\omega$.

Solution. **(A)**

Given that $u = u_0 i + (a\omega\cos\omega t)j$

Hence, velocity along x-axis $u_x = u_0$ .... (1)

Velocity along y-axis $u_y = a\omega\cos\omega t$ .... (2)

We know that $\upsilon = \frac{ds}{dt}$

or $s = \int \upsilon.dt$

So from equations (1) and (2), we have

Displacement at time t in horizontal direction

$x = \int u_0\, dt = u_0 .t$ .... (3)

Displacement in vertical direction

$y = \int a\omega\cos\omega t\, dt = a\sin\omega t$ .... (4)

Eliminating t from equations (3) and (4) we get

$y = a\sin(\omega x/u_0)$ .... (5)

Equation (5) gives the trajectory of the particle.

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EXERCISE

14. The displacement of a particle, moving in a straight line, is given by $s = 2t^2 + 2t + 4$ where s is in metres and t in seconds. The acceleration of the particle is

(A) 2 m/s^2 (B) 4 m/s^2

(C) 6 m/s^2 (D) 8 m/s^2

15. The position x of a particle varies with time t as $x = at^2 - bt^3$. The acceleration of the particle will be zero at time t equal to

(A) $\frac{a}{b}$ (B) $\frac{2a}{3b}$

(C) $\frac{a}{3b}$ (D) Zero

16. The displacement of the particle is given by $y = a + bt + ct^2 - dt^4$. The initial velocity and acceleration are respectively

(A) b, –4d (B) –b, 2c

(C) b, 2c (D) 2c, –4d

17. The relation between time t and distance x is $t = \alpha x^2 + \beta x$, where a and b are constants. The retardation is (v is the velocity)

(A) $2\alpha v^3$ (B) $2\beta v^3$

(C) $2\alpha\beta v^3$ (D) $2\beta^2 v^3$

18. If displacement of a particle is directly proportional to the square of time. Then particle is moving with

(A) Uniform acceleration

(B) Variable acceleration

(C) Uniform velocity

(D) Variable acceleration but uniform velocity.

19. A particle is moving eastwards with velocity of 5 m/s. In 10 sec the velocity changes to 5 m/s northwards. The average acceleration in this time is

(A) Zero (B) $\frac{1}{\sqrt{2}} m/s^2$ toward north-west

(C) toward north-east (D) toward north-west

20. A body starts from the origin and moves along the x-axis such that velocity at any instant is given by ($4t^3 - 2t$), where t is in second and velocity is in m/s. What is the acceleration of the particle, when it is 2m from the origin?

(A) 28 m/s^2 (B) 22 m/s^2

(C) 12 m/s^2 (D) 10 m/s^2

21. A body of mass 10 kg is moving with a constant velocity of 10 m/s. When a constant force acts for 4 sec on it, it moves with a velocity 2 m/sec in the opposite direction. The acceleration produced in it is

(A) 3 m/s^2 (B) –3m/s^2

(C) 0.3 m/s^2 (D) –0.3 m/s^2

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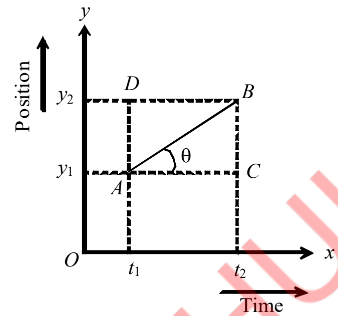
POSITION TIME GRAPH

During motion of the particle its parameters of kinematical analysis (u, v, a, r) changes with time. This can be represented on the graph.

Position time graph is plotted by taking time t along x-axis and position of the particle on y-axis.

Let AB is a position-time graph for any moving particle



As Velocity = $\dfrac{\text{Change in position}}{\text{Time taken}} = \dfrac{y_2 - y_1}{t_2 - t_1}$ ...(1)

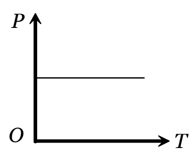
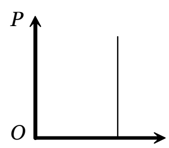
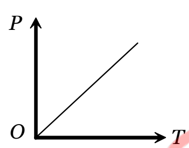
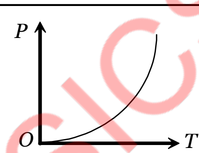
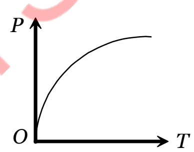
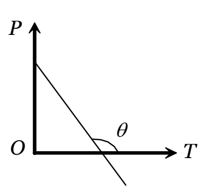
From triangle ABC $\tan\theta = \dfrac{BC}{AC} = \dfrac{AD}{AC} = \dfrac{y_2 - y_1}{t_2 - t_1}$ ...(2)

By comparing (i) and (ii) Velocity = $\tan\theta$

$v = \tan\theta$

It is clear that slope of position-time graph represents the velocity of the particle.

VARIOUS POSITION – TIME GRAPHS AND THEIR INTERPRETATION

	$\theta = 0^\circ$ so $v = 0$ *i.e.*, line parallel to time axis represents that the particle is at rest.
	$\theta = 90^\circ$ so $v = \infty$ *i.e.*, line perpendicular to time axis represents that particle is changing its position but time does not changes it means the particle possesses infinite velocity. Practically this is not possible.
	θ = constant so v = constant, $a = 0$ *i.e.*, line with constant slope represents uniform velocity of the particle
	θ is increasing so v is increasing, a is positive. *i.e.*, line bending towards position axis represents increasing velocity of particle. It means the particle possesses acceleration.
	θ is decreasing so v is decreasing, a is negative *i.e.*, line bending towards time axis represents decreasing velocity of the particle. It means the particle possesses retardation.
	θ constant but > 90° so v will be constant but negative *i.e.*, line with negative slope represent that particle returns towards the point of reference. (negative displacement).

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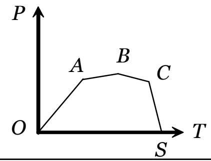
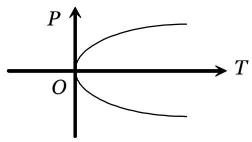
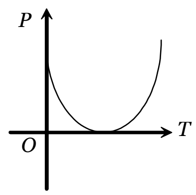
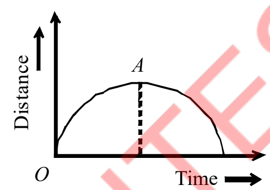
	Straight line segments of different slopes represent that velocity of the body changes after certain interval of time.
	This graph shows that at one instant the particle has two positions. Which is not possible.
	The graph shows that particle coming towards origin initially and after that it is moving away from origin.

Note : If the graph is plotted between distance and time then it is always an increasing curve and it never comes back towards origin because distance never decrease with time. Hence such type of distance time graph is valid up to point A only, after point A it is not valid as shown in the figure.



For two particles having displacement time graph with slopes θ_1 and θ_2 possesses velocities v_1 and v_2 respectively then $\frac{\upsilon_1}{\upsilon_2} = \frac{\tan\theta_1}{\tan\theta_2}$

SOLVED EXAMPLE

Example 19. The position of a particle moving along the x-axis at certain times is given below :

t (*s*)	0	1	2	3
x (*m*)	– 2	0	6	16

Which of the following describes the motion correctly

(A) Uniform, accelerated

(B) Uniform, decelerated

(C) Non-uniform, accelerated

(D) There is not enough data for generalisation

Solution. **(A)**

Instantaneous velocity $v = \frac{\Delta x}{\Delta t}$, By using the data from the table

$v_1 = \frac{0-(-2)}{1} = 2\text{m/s}$, $v_2 = \frac{6-0}{1} = 6\text{m/s}$ and $v_3 = \frac{16-6}{1} = 10\text{ m/s}$ i.e. the speed is increasing at a constant rate so motion is uniformly accelerated.

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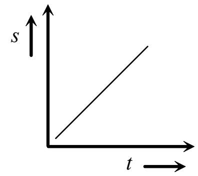
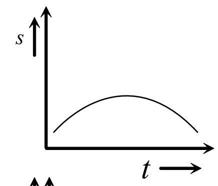
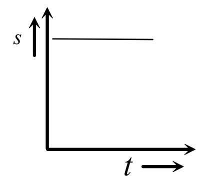
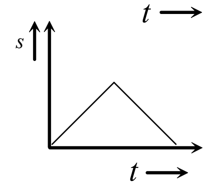
Example 20. Which of the following graph represents uniform motion

(A) 

(B) 

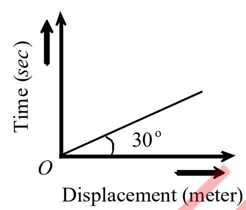
(C) 

(D) 

Solution. **(A)**

When distance time graph is a straight line with constant slope than motion is uniform.

Example 21. From the following displacement time graph find out the velocity of a moving body



(A) $\frac{1}{\sqrt{3}}$ m/s (B) 3 m/s

(C) $\sqrt{3}$ m/s (D) $\frac{1}{3}$

Solution. **(C)**

In first instant you will apply $\upsilon = \tan\theta$ and say, $\upsilon = \tan 30^o = \frac{1}{\sqrt{3}}$ m/s.

But it is wrong because formula is valid when angle is measured with time axis.

Here angle is taken from displacement axis. So angle from time axis .

Now

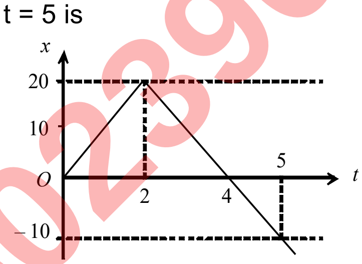
EXERCISE

22. The displacement-time graph for two particles A and B are straight lines inclined at angles of 30° and 60° with the time axis. The ratio of velocities of $v_A : v_B$ is

(A) 1 : 2 (B) $1 : \sqrt{3}$

(C) $\sqrt{3} : 1$ (D) 1 : 3

23. The diagram shows the displacement-time graph for a particle moving in a straight line. The average velocity for the interval t = 0, t = 5 is



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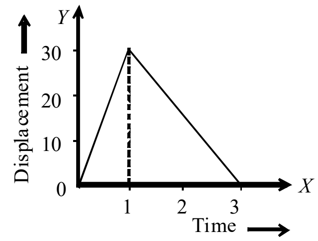
(A) 0 (B) $6\ ms^{-1}$

(C) $-2\ ms^{-1}$ (D) $2\ ms^{-1}$

24. Figure shows the displacement time graph of a body. What is the ratio of the speed in the first second and that in the next two seconds



(A) 1 : 2 (B) 1 : 3

(C) 3 : 1 (D) 2 : 1

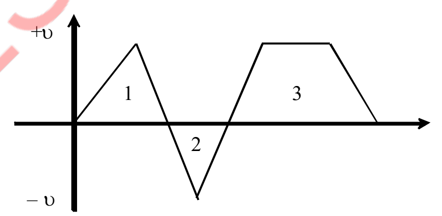
Velocity Time Graph

The graph is plotted by taking time t along x-axis and velocity of the particle on y-axis.

Distance and displacement : The area covered between the velocity time graph and time axis gives the displacement and distance travelled by the body for a given time interval.

Then Total distance = $|A_1| + |A_2| + |A_3|$

= Addition of modulus of different area.



i.e. $s = \int |\upsilon|\, dt$

Total displacement = $A_1 + A_2 + A_3$

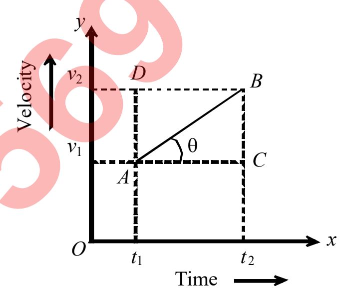
= Addition of different area considering their sign.

i.e. $r = \int \upsilon\, dt$

here A_1 and A_2 are area of triangle 1 and 2 respectively and A_3 is the area of trapezium .

Acceleration : Let AB is a velocity-time graph for any moving particle

As Acceleration = $\dfrac{\text{Change in velocity}}{\text{Time taken}} = \dfrac{v_2 - v_1}{t_2 - t_1}$ ...(i)



From triangle ABC, $\tan\theta = \dfrac{BC}{AC} = \dfrac{AD}{AC} = \dfrac{v_2 - v_1}{t_2 - t_1}$ ....(ii)

By comparing (i) and (ii)

Acceleration (A) = $\tan\theta$

It is clear that slope of velocity-time graph represents the acceleration of the particle.

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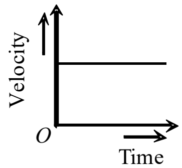
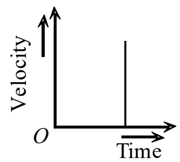
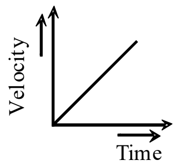
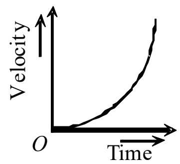
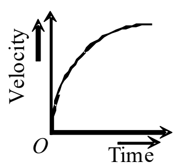
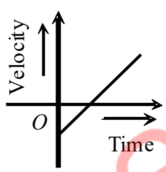
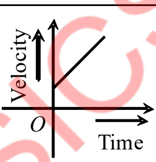
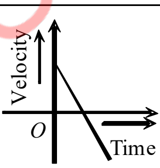
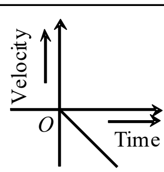
VARIOUS VELOCITY – TIME GRAPHS AND THEIR INTERPRETATION

	θ = 0, a = 0, v = constant i.e., line parallel to time axis represents that the particle is moving with constant velocity.
	θ = 90°, a = ∞, v = increasing i.e., line perpendicular to time axis represents that the particle is increasing its velocity, but time does not change. It means the particle possesses infinite acceleration. Parctically it is not possible.
	θ = constant, so a = constant and v is increasing uniformly with time i.e., line with constant slope represents uniform acceleration of the particle
	θ is increasing so acceleration increasing i.e., line bending towards velocity axis represent the increasing acceleration in the body
	θ decreasing so acceleration decreasing i.e. line bending towards time axis represent the decreasing acceleration in the body.
	Positive constant acceleration but θ is constant and < 90° but initial velocity of the particle is negative.
	Positive constant acceleration but θ is constant and > 90° but initial velocity of particle is positive
	Negative constant acceleration because θ is constant and > 90° but initial velocity of the particle is positive.
	Negative constant acceleration because θ is constant and > 90° but initial velocity of the particle is zero.

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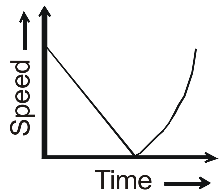
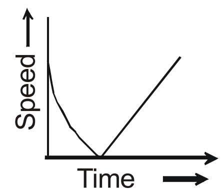
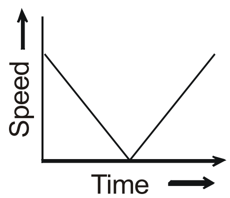
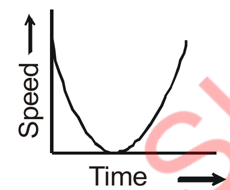
SOLVED EXAMPLE

Example 22. A ball is thrown vertically upwards. Which of the following plots represents the speed-time graph of the ball during its flight if the air resistance is not ignored

(A)


(B)


(C)


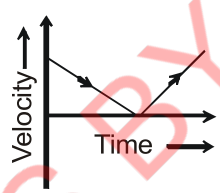
(D)


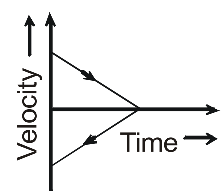
Solution. **(C)**

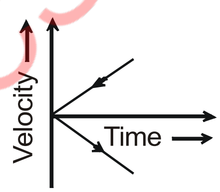
In first half of motion the acceleration is uniform & velocity gradually decreases, so slope will be negative but for next half acceleration is positive. So slope will be positive. Thus graph 'C' is correct.

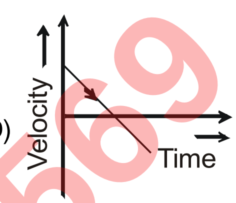
Not ignoring air resistance means upward motion will have acceleration (a + g) and the downward motion will have (g – a).

Example 22. A ball is thrown vertically upward which of the following graph represents velocity time graph of the ball during its flight (air resistance is neglected)

(A)


(B)


(C)


(D)


Solution. **(D)**

In the positive region the velocity decreases linearly (during rise) and in negative region velocity increase linearly (during fall) and the direction is opposite to each other during rise and fall, hence fall is shown in the negative region.

Example 23. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $\frac{d}{2}$. Neglecting subsequent motion and air resistance, its velocity υ varies with the height h above the ground as.

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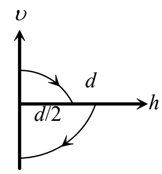
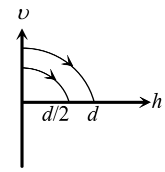
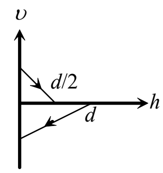
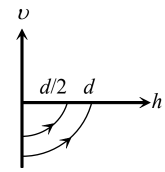
(A) 

(B) 

(C) 

(D) 

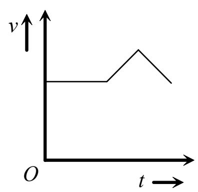
Solution. **(A)**

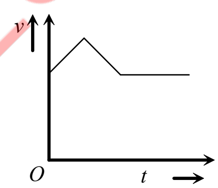
When ball is dropped from height d its velocity will be zero.

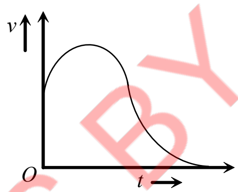
As ball comes downward h decreases and υ increases just before the rebound from the earth h = 0 and v = maximum and just after rebound velocity reduces to half and direction becomes opposite. As soon as the height increases its velocity decreases and becomes zero at $h = \frac{d}{2}$.

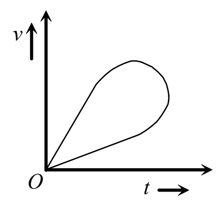
This interpretation is clearly shown by graph (A).

Example 24. Which of the following velocity time graphs is not possible

(A) 

(B) 

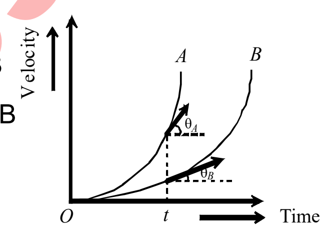
(C) 

(D) 

Solution. **(D)**

Particle can not possess two velocities at a single instant so graph (D) is not possible.

Example 25. Velocity-time graphs of two cars which start from rest at the same time, are shown in the figure. Graph shows, that



(A) Initial velocity of A is greater than the initial velocity of B

(B) Acceleration in A is increasing at lesser rate than in B

(C) Acceleration in A is greater than in B

(D) Acceleration in B is greater than in A

Solution. **(C)**

At a certain instant t slope of A is greater than B ($\theta_A > \theta_B$), so acceleration in A is greater than B

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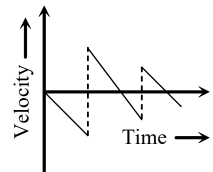
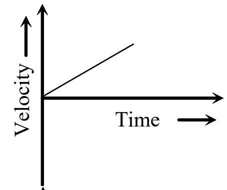
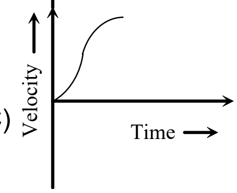
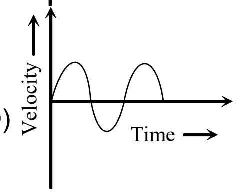
Example 26. Which one of the following graphs represent the velocity of a steel ball which fall from a height on to a marble floor? (Here υ represents the velocity of the particle and t the time)

(A)


(B)


(C)


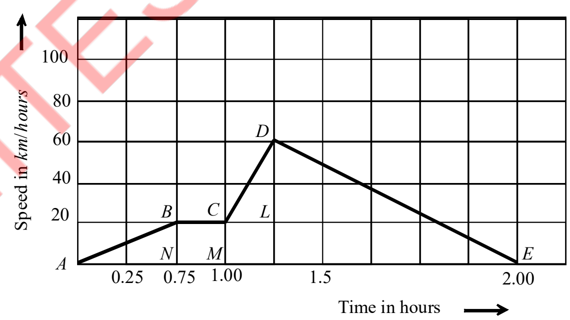
(D)


Solution. **(A)**

Initially when ball falls from a height its velocity is zero and goes on increasing when it comes down. Just after rebound from the earth its velocity decreases in magnitude and its direction gets reversed. This process is repeated untill ball comes to at rest. This interpretation is well explained in graph (A).

EXERCISE

25. A train moves from one station to another in 2 hours time. Its speed-time graph during this motion is shown in the figure. The maximum acceleration during the journey is



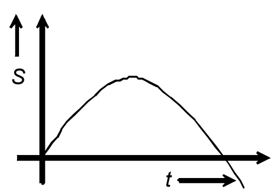
(A) 140 km h^{-2}

(B) 160 km h^{-2}

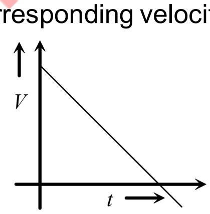
(C) 100 km h^{-2}

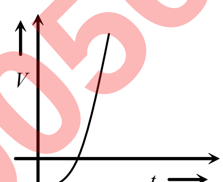
(D) 120 km h^{-2}

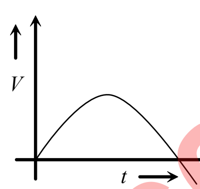
26. The graph of displacement v/s time is

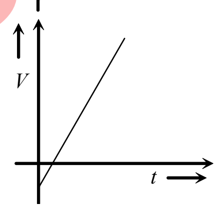


Its corresponding velocity-time graph will be

(A)


(B)


(C)


(D)


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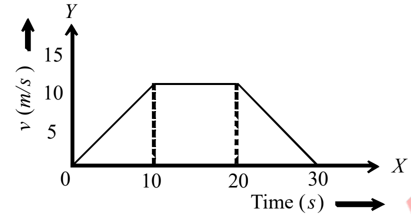
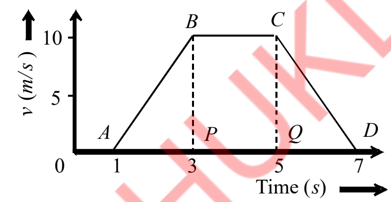
27. In the following graph, distance travelled by the body in metres is



(A) 200 (B) 250

(C) 300 (D) 400

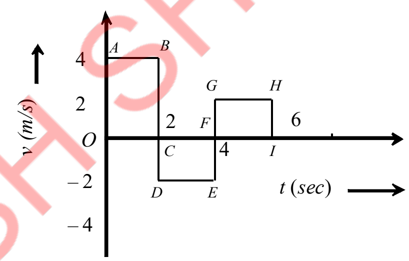
28. For the velocity-time graph shown in figure below the distance covered by the body in last two seconds of its motion is what fraction of the total distance covered by it in all the seven seconds



(A) $\frac{1}{2}$ (B) $\frac{1}{4}$

(C) $\frac{1}{3}$ (D) $\frac{2}{3}$

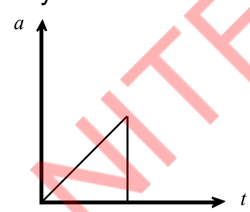
29. The velocity time graph of a body moving in a straight line is shown in the figure. The displacement and distance travelled by the body in 6 sec are respectively



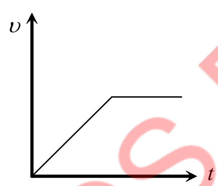
(A) 8 m, 16 m (B) 16 m, 8 m

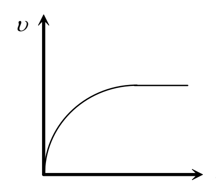
(C) 16 m, 16 m (D) 8 m, 8 m

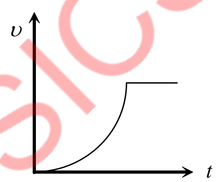
30. The acceleration-time graph of a body is shown below –

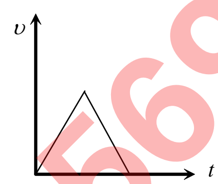


The most probable velocity-time graph of the body is

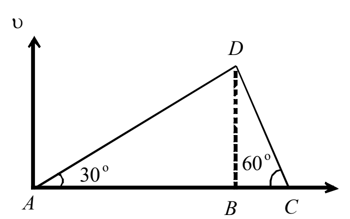
(A)


(B)


(C)


(D)


31. For a certain body, the velocity-time graph is shown in the figure. The ratio of applied forces for intervals AB and BC is



(A) $+\frac{1}{2}$ (B) $-\frac{1}{2}$

(C) $+\frac{1}{3}$ (D) $-\frac{1}{3}$

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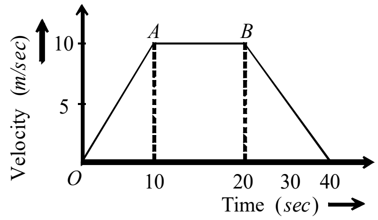
32. The adjoining curve represents the velocity-time graph of a particle, its acceleration values along OA, AB and BC in metre/sec^2 are respectively

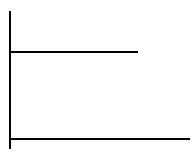
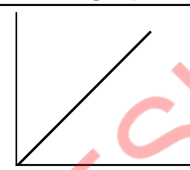
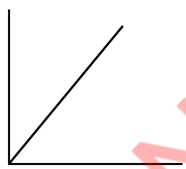
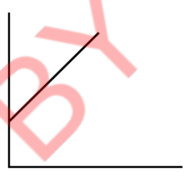
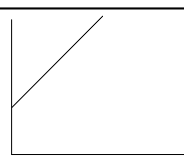
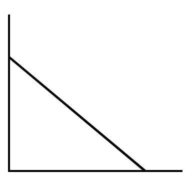
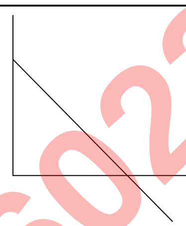


(A) 1, 0, – 0.5 (B) 1, 0, 0.5

(C) 1, 1, 0.5 (D) 1, 0.5, 0

SOME IMPORTANT GRAPHS

All the following graphs are drawn for one-dimensional motion with uniform velocity or with constant acceleration.

Different case	v-t graph	s-t graph	Important Point
1. Uniform motion			(i) Slope of s-t graph = v = constant (ii) In s-t graph s = 0 at t = 0
2. Uniformly accelerated motion with u = 0 and s = 0 at t = 0			(i) u = 0, ie., v = 0 at t = 0 (ii) u = 0, i.e, slope of s-t graph at t = 0, should be zero (iii) a or slope of v-t graph is constant.
3. Uniformly accelerated motion with $u \neq 0$ but s = 0 at t = 0			(i) $u \neq 0$, i.e., v or slope of s-t graph at t = 0 is not zero. (ii) v or slope of s-t graph gradually goes on increasing.
4. Uniformly accelerated motion with $u \neq 0$ and $s = s_0$ at t = 0			(i) $s = s_0$ at t = 0
5. Uniformly retarded motion till velocity becomes zero.			(i) slope of s-t graph at t = 0 gives u (ii) slope of s-t graph at $t = t_0$ becomes zero. (iii) In this case u can't be zero.
6. Uniformly retarded then accelerated in opposite direction			(i) At time $t = t_0$, v = 0 or slope of s-t graph is zero. (ii) In s-t graph slope or velocity first decreases then increases with opposite sign.

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Equations of Kinematics/Motion gives

Where the various relations between u, v, a, t and s for the moving particle are given below.:

u = Initial velocity of the particle at time t = 0 sec

v = Final velocity at time t sec

a = Acceleration of the particle

s = Distance travelled in time t sec

EQUATIONS OF MOTION

Motion **under uniform acceleration** is described by the following equations.

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

1. The velocity time relation

$a = \frac{dv}{dt}$

$dv = adt$

If the velocity of particle at time t_1 is v_1 and at time t_2 is v_2

$\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a\,dt$

$[v]_{v_1}^{v_2} = a[t]_{t_1}^{t_2}$

$v_2 - v_1 = a[t_2 - t_1]$

$v_2 = v_1 + a[t_2 - t_1]$

If $t_1 = 0, v_1 = u$ and $t_2 = t, v_2 = v$

$v = u + a\,t$

2. Position-time relation

By $v = \frac{dx}{dt}$ or $dx = vdt$

$\Rightarrow \int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v\,dt$

$\Rightarrow \int_{x_1}^{x_2} dx = \int_0^t (u + at)\,dt$

$\Rightarrow [x]_{x_1}^{x_2} = \left[ut + \frac{1}{2}at^2\right]_0^t$

$x_2 - x_1 = ut + \frac{1}{2}at^2$

$s = ut + \frac{1}{2}at^2$

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3. Velocity-position relation

$$a = \frac{dv}{dt} = \frac{dv}{dt}.\frac{dx}{dx} = v\frac{dv}{dx} \quad \text{or } a\ dx = v\ dv$$

$$\int_{x_1}^{x_2} a\,dx = \int_{v_1}^{v_2} v\,dv$$

$$a[x]_{x_1}^{x_2} = \left[\frac{v^2}{2}\right]_{v_1}^{v_2}$$

$$2a(x_2 - x_1) = v_2^2 - v_1^2$$

$$v^2 - u^2 = 2as$$

$$v^2 = u^2 + 2as$$

4. Distance travelled in n^{th} second of uniformly accelerated motion :

$$S_{n^{th}} = u + \frac{a}{2}(2n - 1)$$

The calculation of speed and distance by acceleration-time graph:

Let a particle be moving with uniform acceleration according to following a–t graph – dv = a dt

or $$\int_u^v dv = \int_{t_1}^{t_2} a\,dt$$

$$(v)_u^v = a(t)_{t_1}^{t_2}$$

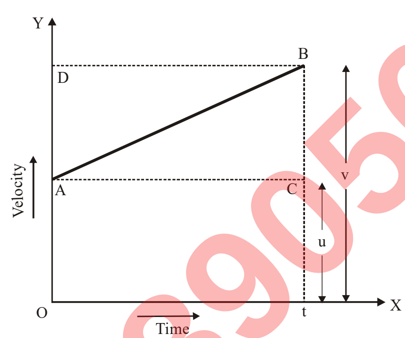
$$v - u = a(t_2 - t_1)$$

Therefore difference in magnitude of velocity (v – u) = AB x AD

v – u = Area of rectangle ABCD = area under a – t graph

Equations of Motion (Graphical Method)

Consider a body having initial velocity '**u**'. Suppose it is subjected to a uniform acceleration '**a**' so that after time'**t**' its final velocity becomes '**v**' shown by points **A** & **B** on the graph correspond to the times **0** and **t** respectively.



First Equation of Motion

The slope of the velocity-time graph gives the acceleration of a particle moving in a straight line.

For line AB,

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$$\text{Slope} = \frac{BC}{AC} \text{ (By defination)}$$

$$a = \frac{BC}{AC} = \frac{BE - CE}{OE} = \frac{OD - OA}{OE}$$

$$a = \frac{v - u}{t}$$

$$v = u + a$$

Second Equation of Motion

The area under the velocity-time graph is equal to the displacement.

In the time interval 0 – t, displacement = area OABE

s = area OABE=area of the rectangle OACE + area of the triangle ABC

$$s = (OA).(OE) + \frac{1}{2}(AC).(BC) = (OA).(OE) + \frac{1}{2}(AC).(BE - EC)$$

$$s = u.t + \frac{1}{2}t(v - u)$$

$$s = u.t + \frac{1}{2}t.at \qquad \left(\therefore a = \frac{v - u}{t} \qquad 1^{st} \text{ equation of motion}\right)$$

$$s = ut + \frac{1}{2}at^2$$

Third Equation of Motion

Displacement = The area under the v-t graph is =area of trapezium **AOEB**

area of trapezium = $\frac{1}{2}$ (sum of parallel sides) × (perpendicular distance)

$$s = \frac{1}{2}(OA + EB).(AC)$$

$$= \frac{1}{2}(OA + OD).(AC)$$

$$= \frac{1}{2}(u + v).(t)$$

$$= \frac{1}{2}(v + u).\left(\frac{v - u}{a}\right) \quad (\because v = u + at \quad 1^{st} \text{ equation of motion})$$

$$s = \frac{1}{2}\left(\frac{v^2 - u^2}{a}\right)$$

$$v^2 - u^2 = 2as$$

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(1) When particle moves with zero acceleration

(i) It is a unidirectional motion with constant speed.

(ii) Magnitude of displacement is always equal to the distance travelled.

(iii) v = u, s = u t [As a = 0]

(2) When particle moves with constant acceleration

(i) Acceleration is said to be constant when both the magnitude and direction of acceleration remain constant.

(ii) There will be one dimensional motion if initial velocity and acceleration are parallel or anti-parallel to each other.

(iii) Equations of motion in scalar from Equation of motion in vector from

$v = u + at$	$\vec{v} = \vec{u} + \vec{a}t$
$s = ut + \frac{1}{2}at^2$	$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
$v^2 = u^2 + 2as$	$\vec{v}.\vec{v} - \vec{u}.\vec{u} = 2\vec{a}.\vec{s}$
$s = \left(\frac{u+v}{2}\right)t$	
$\vec{s} = \frac{1}{2}(\vec{u} + \vec{v})t$	
$s_n = u + \frac{a}{2}(2n-1)$	$\vec{s}_n = \vec{u} + \frac{\vec{a}}{2}(2n-1)$

(3) Important points for uniformly accelerated motion

i. If a body starts from rest and moves with uniform acceleration then distance covered by the body in t sec is proportional to t^2 (i.e. $s \propto t^2$).

So we can say that the ratio of distance covered in 1 sec, 2 sec and 3 sec is $1^2 : 2^2 : 3^2$ or 1 : 4 : 9.

ii. If a body starts from rest and moves with uniform acceleration then distance covered by the body in nth sec is proportional to (2n – 1)(i.e. $s_n \propto (2n-1)$

So we can say that the ratio of distance covered in I sec, II sec and III sec is 1 : 3 : 5.

iii. A body moving with a velocity u is stopped by application of brakes after covering a distance s. If the same body moves with velocity nu and same braking force is applied on it then it will come to rest after covering a distance of n^2s.

As $v^2 = u^2 - 2as \Rightarrow 0 = u^2 - 2as \Rightarrow s = \frac{u^2}{2a}$ $s \propto u^2$ [since a is constant]

So we can say that if u becomes n times then s becomes n^2 times that of previous value.

iv. A particle moving with uniform acceleration from A to B along a straight line has velocities υ_1 and v_2 at A and B respectively. If C is the mid-point between A and B then velocity of the particle at C is equal to

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SOLVED EXAMPLE

Example 27. The velocity acquired by a body moving with uniform acceleration is 20 m/s in first 2 sec and 40 m/s in first 4 sec. Calculate initial velocity.

Solution. $a = \frac{v_2 - v_1}{t_2 - t_1}$

$$a = \frac{40-20}{4-2} = \frac{20}{2} = 10\,m/s^2$$

Now, $v = u + at$

$$v_1 = u + at_1$$

$$\Rightarrow \quad 20 = u + 10 \times 2$$

$$\Rightarrow \quad 20 = u + 20 \quad \Rightarrow \quad u = 0\,m/s$$

Example 27. A particle starts with initial velocity 2.5 m/s along the x direction and accelerates uniformly at the rate 50 cm/s^2. Find time taken to increase the velocity to 7.5 m/s.

Solution. $v = u + at$

$$7.5 = 2.5 + 0.5t$$

$$5.0 = 0.5t$$

$$t = \frac{50}{5} = 10\,sec$$

Example 28. A particle starts with a constant acceleration. At a time t second speed is found to be 100 m/s and one second later speed becomes 150 m/s. Find acceleration of the particle.

Solution. From equation (1) of motion $v = u + at$

$$\Rightarrow \quad 100 = 0 + at$$

$$100 = at \quad \text{... (1)}$$

Now consider velocity one second later -

$$v' = 0 + a(t+1)$$

$$\Rightarrow \quad 150 = a(t+1) \quad \text{... (2)}$$

On subtracting equation (1) from equation (2)

$$a = 50\,m/s^2$$

Example 29. A truck starts from rest with an acceleration of 1.5 ms^{-2} while a car 150 metre behind starts from rest with an acceleration of 2 ms^{-2}. (A) How long will it take before both the truck and car are side by side and (B) How much distance is travelled by each.

Solution. **(A)** $s_T = \frac{1}{2}at^2$

$$s_T = \frac{1}{2}(1.5)t^2 \quad \text{.... (1)}$$

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Distance covered by car when car one overtakes the truck

$$s_c = \frac{1}{2}(2)t^2$$

$$(s_T + 150) = \frac{1}{2}(2)t^2 \qquad .... (2)$$

Divide equation (2) by equation (1) $\frac{s_T + 150}{s_T} = \frac{2}{1.5} \Rightarrow 1 + \frac{150}{s_T} = \frac{20}{15} = \frac{4}{3}$

$$\Rightarrow \quad \frac{150}{s_T} = \frac{4}{3} - 1 = \frac{1}{3} \text{ or } \quad s_T = 450$$

Distance travelled by car = 450 + 150 = 600 metre

(B) Now by equation (1) $s_T = \frac{1}{2}at^2$

$$450 = \frac{1}{2} \times 1.5 \times t^2$$

$$t^2 = \frac{450 \times 2}{1.5} \quad \Rightarrow \quad t = \sqrt{300 \times 2} = 24.5\,\text{sec}$$

Therefore car will overtake the truck after 24.5 second.

Example 30. A body travels a distance of 2 m in 2 seconds and 2.2 m in next 4 seconds. What will be the velocity of the body at the end of 7th second from the start.

Solution. Here, case (i) s = 2m, t = 2s

case (ii) t = 2 + 4 = 6s

Let u and a be the initial velocity and uniform acceleration of the body.

we know that, $s = ut + \frac{1}{2}at^2$

Case (i) $2 = u \times 2 + \frac{1}{2}a \times 2^2$

or 1 = u + a .... (1)

Case (ii) $4.2 = u \times 6 + \frac{1}{2}a \times 6^2$

or $0.7 = u + 3a$ ... (2)

Subtracting (2) from (1), we get

$0.3 = 0 - 2a = -2a$

or a = – 0.3/2 = –0.15 ms^{-2}

From (i), $u = 1 - a = 1 + 0.15$

$u = 1.15\,ms^{-1}$

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For the velocity of body at the end of 7th second, we have

$u = 1.15 ms^{-1}; a = -0.15 ms^{-2}$,

v = ?, t = 7s

As, v = u + at

$\therefore$ v = 1.15 + (–0.15) x 7

v = 0.1 m/s

Example 31. A body travels a distance of 20 m in the 7th second and 24 m in 9th second. How much distance shall it travel in the 15th second?

Solution. Here, s_7 = 20 m ; s_9 = 24 m,

s_{15} = ?

Let u and a be the initial velocity and uniform acceleration of the body.

We know that, $s_n = u + \frac{a}{2}(2n-1)$

$\therefore \; s_7 = u + \frac{a}{2}(2 \times 7 - 1)$

or $20 = u + \frac{13a}{2}$ .... (i)

and $s_9 + u + \frac{a}{2}(2 \times 9 - 1)$

or 24 = u + $\frac{17}{2}$ a .... (ii)

Subtracting (ii) form (i), we get

4 = 2a

or a = 2 ms^{-2}

Putting this value in (i), we get

$20 = u + \frac{13}{2} \times 2$ or 20 = u + 13

or u = 20 – 13 = 7 ms^{-1}

Hence, s_{15} = $u + \frac{a}{2}$ (2 × 15 – 1) = 7 + $\frac{2}{2}$ × 29

s_{15} = 36 m

Example 32. A person travelling at 43.2 km/h applies the brakes giving a deceleration of 6 m/s^2 to his scooter. How far will it travel before stopping?

Solution. Here, $u = 43.2 km/h$ = $43.2 \times \frac{5}{18} m/s$

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Deceleration; a = 6 m/s^2

v = 0 s = ?

Using $v^2 = u^2 - 2as$

$0 = (12)^2 - 2 \times 6\ s$

144 = 2 x 6s

$$s = \frac{144}{12} = 12m$$

Example 33. A bullet going with speed 350 m/s enters in a concrete wall and penetrates a distance of 5 cm before coming to rest. Find deceleration.

Solution. Here, u = 350 m/s, s = 5 cm,

v = 0 m/s and a = ?

By using $v^2 = u^2 - 2as$

$$0 = u^2 - 2as$$

$$u^2 = 2as \quad \text{or} \quad a = \frac{u^2}{2s}$$

$$a = \frac{350 \times 350}{2 \times 0.05} = 12.25 \times 10^5 \text{ m/sec}^2$$

Example 34. A driver takes 0.20 s to apply the brakes after he sees a need for it. This is called the reaction time of the driver. If he is driving a car at a speed of 54 km/h and the brakes cause a deceleration of 6.0 m/s^2, find the distance travelled by the car after he seeds the need to put the brakes on

Solution. Distance covered by the car during the application of brakes by driver -

$$u = 54 \text{ km/h} = 54 \times \frac{5}{18} \text{m/s} = 15 \text{ m/s}$$

$s_1 = ut$ or $s_1 = 15 \times 0.2 = 3.0$ meter

After applying the brakes;

v = 0 u = 15 m/s, a = 6 m/s^2

s_2 = ?

Using $v^2 = u^2 - 2as$

$0 = (15)^2 - 2 \times 6 \times s_2$

$12\ s_2 = -225$

$$\Rightarrow s_2 = \frac{225}{12} = 18.75 \text{metre}$$

Distance travelled by the car after driver sees the need for it

$s = s_1 + s_2$

s = 3 + 18.75 = 21.75 metre.

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EXERCISE

33. A body A moves with a uniform acceleration a and zero initial velocity. Another body B, starts from the same point moves in the same direction with a constant velocity v. The two bodies meet after a time t. The value of t is

(A) $\frac{2v}{a}$ (B) $\frac{v}{a}$

(C) $\frac{v}{2a}$ (D) $\sqrt{\frac{v}{2a}}$

34. A student is standing at a distance of 50 metres from the bus. As soon as the bus starts its motion with an acceleration of $1 ms^{-2}$, the student starts running towards the bus with a uniform velocity u. Assuming the motion to be along a straight road, the minimum value of u, so that the students is able to catch the bus is

(A) $5\ ms^{-1}$ (B) $8\ ms^{-1}$

(C) $10\ ms^{-1}$ (D) $12\ ms^{-1}$

35. A car, moving with a speed of 50 km/hr, can be stopped by brakes after at least 6m. If the same car is moving at a speed of 100 km/hr, the minimum stopping distance is

(A) 6m (B) 12m

(C) 18m (D) 24m

36. The velocity of a bullet is reduced from 200m/s to 100m/s while travelling through a wooden block of thickness 10cm. The retardation, assuming it to be uniform, will be

(A) $10 \times 10^4\ m/s^2$ (B) $12 \times 10^4\ m/s^2$

(C) $13.5 \times 10^4\ m/s^2$ (D) $15 \times 10^4\ m/s^2$

37. A body A starts from rest with an acceleration a_1. After 2 seconds, another body B starts from rest with an acceleration a_2. If they travel equal distances in the 5th second, after the start of A, then the ratio $a_1 : a_2$ is equal to

(A) 5 : 9 (B) 5 : 7

(C) 9 : 5 (D) 9 : 7

38. The average velocity of a body moving with uniform acceleration travelling a distance of 3.06 m is $0.34\ ms^{-1}$. If the change in velocity of the body is $0.18 ms^{-1}$ during this time, its uniform acceleration is

(A) $0.01\ ms^{-2}$ (B) $0.02\ ms^{-2}$

(C) $0.03\ ms^{-2}$ (D) $0.04\ ms^{-2}$

39. A particle travels 10m in first 5 sec and 10m in next 3 sec. Assuming constant acceleration what is the distance travelled in next 2 sec

(A) 8.3 m (B) 9.3 m

(C) 10.3 m (D) None of above

40. A body travels for 15 sec starting from rest with constant acceleration. If it travels distances and in the first five seconds, second five seconds and next five seconds respectively the relation between and is

(A) $S_1 = S_2 = S_3$ (B) $5S_1 = 3S_2 = S_3$

(C) $S_1 = \frac{1}{3}S_2 = \frac{1}{5}S_3$ (D) $S_1 = \frac{1}{5}S_2 = \frac{1}{3}S_3$

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41. If a body having initial velocity zero is moving with uniform acceleration the distance travelled by it in fifth second will be

(A) 36 metres (B) 40 metres

(C) 100 metres (D) Zero

42. The engine of a car produces acceleration $4 m/sec^2$ in the car, if this car pulls another car of same mass, what will be the acceleration produced

(A) $8 m/s^2$ (B) $2 m/s^2$

(C) $4 m/s^2$ (D) $16 m/s^2$

43. A body starts from rest. What is the ratio of the distance travelled by the body during the 4^{th} and 3^{rd} second.

(A) 7/5 (B) 5/7

(C) 7/3 (D) 3/7

MOTION OF THE BODY UNDER GRAVITY (FREE FALL).

The force of attraction of earth on bodies, is called force of gravity. Acceleration produced in the body by the force of gravity, is called acceleration due to gravity. It is represented by the symbol g.

In the absence of air resistance, it is found that all bodies (irrespective of the size, weight or composition) fall with the same acceleration near the surface of the earth. This motion of a body falling towards the earth from a small altitude (h << R) is called free fall.

An ideal one-dimensional motion under gravity in which air resistance and the small changes in acceleration with height are neglected

(1) Body falling freely under gravity :

Taking initial position as origin and downward direction of motion as positive, we have

$u = 0$ [as body starts from rest]

$a = +g$ [as acceleration is in the direction of motion]

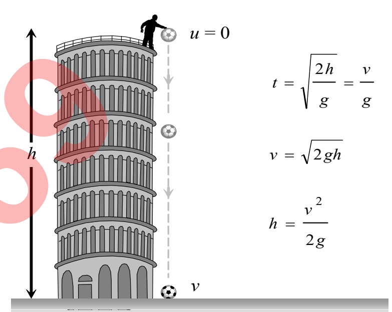
So, if the body acquires velocity v after falling a distance h in time t, equations of motion, viz.

$v = u + at$; $s = ut + \frac{1}{2}at^2$ and $v^2 = u^2 + 2as$

reduces to $v = gt$ ... (1)

$h = \frac{1}{2}gt^2$ ... (2)

and $v^2 = 2gh$ .... (3)



These equations can be used to solve most of the problems of freely falling bodies as if.

t can be derived
From eqns. (1) and (2)

h can be derived
From eqns. (2) and (3)

V can be derived
From eqns. (3) and (1)

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$v = gt$

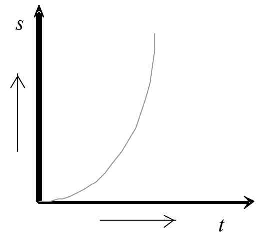
and $h = \frac{1}{2}gt^2$

$t = \sqrt{\frac{2h}{g}}$

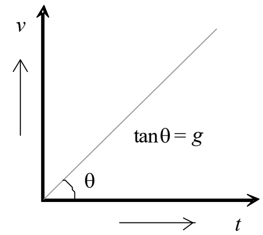
$v = \sqrt{2gh}$

$t = \frac{v}{g}$

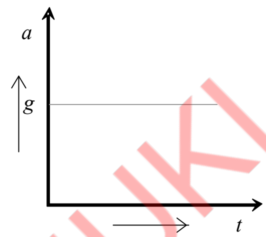
$h = \frac{v^2}{2g}$



(A)



(B)



(C)

(i) If the body is dropped from a height H, as in time t is has fallen a distance h from its initial position, the height of the body from the ground will be

$$h' = H - h \text{ with } h = \left(\frac{1}{2}\right)gt^2$$

(ii) As $h = \left(\frac{1}{2}\right)gt^2$, i.e., $h \propto t^2$, distance fallen in time, t,2t,3t etc., will be in the ratio of $1^2 : 2^2 : 3^2$, i.e., square of integers.

(iii) The distance fallen in the n^{th} sec,

$$h_{(n)} - h_{(n-1)} = \frac{1}{2}g(n)^2 - \frac{1}{2}g(n-1)^2$$

$$= \frac{1}{2}g(2n-1)$$

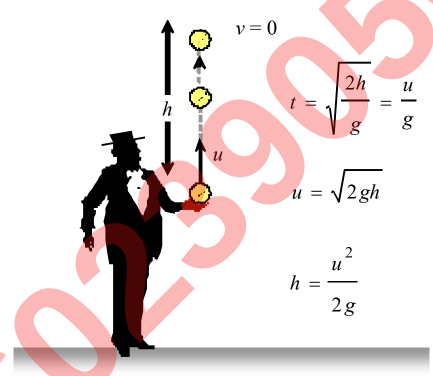
So distances fallen in 1st, 2nd, 3rd sec etc. will be in the ratio of 1 : 3 : 5 i.e., odd integers only.

(2) Body projected vertically up :

Taking initial position as origin and direction of motion (i.e., vertically up) as positive.

here we have v = 0 [at highest point velocity = 0]

a = – g[as acceleration is downwards while motion upwards]



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If the body is projected with velocity u and reaches the highest point at a distance h above the ground in time t, the equations of motion viz.,

$v = u + at$; $s = ut + \frac{1}{2}at^2$ and $v^2 = u^2 + 2as$

reduces to $0 = u - gt$ $h = ut - \frac{1}{2}gt^2$ and $0 = u^2 - 2gh$

Substituting the value of u from first equation in second and rearranging these,

$u = gt$ ...(1)

$h = \frac{1}{2}gt^2$ ...(2)

and $u^2 = 2gh$ ...(3)

These equations can be used to solve most of the problems of bodies projected vertically up as if.

t is given
From eqns. (1) and (2)

$u = gt$

and $h = \frac{1}{2}gt^2$

h is given
From eqns. (2) and (3)

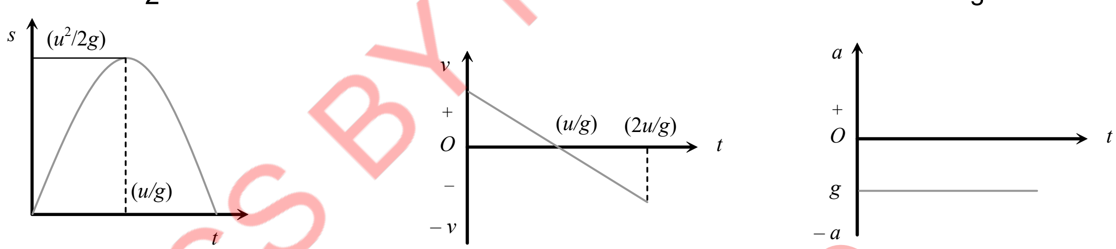
$t = \sqrt{\frac{2h}{g}}$

$v = \sqrt{2gh}$

u is given
From eqns. (3) and (1)

$t = \frac{u}{g}$

$h = \frac{u^2}{2g}$



(A) (B) (C)

Important Points

(1) In case of motion under gravity for a given body, mass, acceleration, and mechanical energy remain constant while speed, velocity, momentum, kinetic energy and potential energy change.

(2) The motion is independent of the mass of the body, as in any equation of motion, mass is not involved. This is why a heavy and lighter body when released from the same height, reach the ground simultaneously and with same velocity.

i.e $t = \sqrt{(2h/g)}$ and $v = \sqrt{2gh}$

However, momentum, kinetic energy or potential energy depend on the mass of the body (all $\propto$ mass)

(3) As from equation (2) time taken to reach a height h,

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$t_u = \sqrt{(2h/g)}$

Similarly, time taken to fall down through a distance h,

$t_D = \sqrt{(2h/g)}$

so $t_u = t_D = \sqrt{(2h/g)}$

So in case of motion under gravity time taken to go up a height h is equal to the time taken to fall down through the same height h.

(4) If a body is projectd vertically up and it reaches a height h, then

$u = \sqrt{(2gh)}$

and if a body falls freely through a height h, then

$v = \sqrt{(2gh)} = u$

So in case of motion under gravity, the speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.

SOLVED EXAMPLE

Example 35. A juggler throws balls into air. He throws one whenever the previous one is at its highest point. How high do the balls rise if he throws n balls each sec. Acceleration due to gravity is g.

Solution. Since the juggler is throwing n balls each second and he throws second ball when the first ball is at the highest point, so time taken by each ball to reach the highest point is t = 1/n

Taking vertical upward motion of ball up to the highest point, we have

$u = 0,\ a = -g,\ t = 1/n,\ u = ?$

As $v = u + at$

so $0 = 0\ u + (-g)\ 1/n$

or $u = g/n$

Also $v^2 = u^2 + 2as,$

so $0 = u^2 - 2gh$

i.e., $h = (u^2/2g) = g/(2n^2)$ (as $u = g/h$)

Example 36. A ball is projected vertically up with an initial speed of 20 m/s on a planet where acceleration due to gravity is 10 $m/s^{2.}$

(A) How long does it takes to reach the highest point?

(B) How high does it rise above the point of projection?

(C) How long will it take for the ball to reach a point 10 m above the point of projection?

Solution. As here motion is vertically upwards,

$a = -g$ and $v = 0$

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(A) From 1st equation of motion, i.e., v = u + at,

$0 = 20 - 10t$

i.e. $t = 2$ sec.

(B) Using $v^2 = u^2 + 2as$

$0 = (20)^2 - 2 \times 10 \times h$

i.e $h = 20$ m.

(C) Using $s = ut + \frac{1}{2}at^2$,

$10 = 20t\ (-\frac{1}{2}) \times 10 \times t^2$

i.e. $t^2 - 4t + 2 = 0$

or $t = 2 \pm \sqrt{2}$,

i.e. $t = 0.59$ sec. or 3.41 sec.

i.e., there are two times, at which the ball passes through h = 10 m, once while going up and then coming down.

Example 37. A ball is thrown vertically upwards from a bridge with an initial velocity of 4.9 m/s. It strikes the water after 2s. If acceleration due to gravity is 9.8 m/s^2 (A) What is the height of the bridge? (B) With which velocity does the ball strike the water?

Solution. Taking the point of projection as origin and downward direction as positive,

(A) Using $s = ut + \left(\frac{1}{2}\right)at^2$ we have

$$h = -4.9 \times 2 + \left(\frac{1}{2}\right)9.8 \times 2^2 = 9.8\,m$$

(u is taken to be negative as it is upwards)

(B) Using $v = u + at$

$v = -4.9 + 9.8 \times 2 = 14.7$ m/s

Example 38. A rocket is fired vertically up from the ground with a resultant vertical acceleration of 10 m/s^2. The fuel is finished in 1minute and it continues to move up.

(A) What is the maximum height reached?

(B) After how much time from then will the maximum height be reached?

(Take g = 10 m/s^2)

Solution. (A) The distance travelled by the rocket during burning interval (1 minute = 60 s) in which resultant acceleration is vertically upwards and 10 m/s^2 will be

$$h_1 = 0 \times 60 + (1/2) \times 10 \times 60^2 = 18000\,m \quad ...(1)$$

Velocity acquired by it is

$v = 0 + 10 \times 60 = 600$ m/s ...(2)

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After one minute the rocket moves vertically up with initial velocity of 600 m/s and continues till height h_2 till its velocity becomes zero.

$$0 = (600)^2 - 2gh_2$$

or $h_2 = 18000\text{m}$ [as g = 10 m/s^2] ...(3)

From equations (1) and (3) the maximum height reached by the rocket from the ground is

$$H = h_1 + h_2 = 18 + 18 = 36\text{km}$$

(B) The time to reach maximum height after burning of fuel is

0 = 600 – gt

t = 60 s

After finishing fuel the rocket goes up for 60 s.

Example 39. A body is released from a height and falls freely towards the earth. Exactly 1 sec later another body is released. What is the distance between the two bodies after 2 sec the release of the second body, if g = 9.8 m/s^2.

Solution. The 2nd body falls for 2s, so

$$h_2 = \frac{1}{2}g(2)^2 \quad \text{...(1)}$$

While 1st has fallen for 2 + 1 = 3 sec so

$$h_1 = \frac{1}{2}g(3)^2 \quad \text{...(2)}$$

∴ Separation between two bodies after 2 sec the release of 2nd body,

$$d = h_1 - h_2$$

$$= \frac{1}{2}g(3^2 - 2^2) = 4.9 \times 5 = 24.5\text{m}$$

Example 40. If a body travels half its total path in the last second of its fall from rest, find : (A) The time and (B) height of its fall. Explain the physically unacceptable solution of the quadratic time equation. (g = 9.8 m/s^2)

Solution. If the body falls a height h in time t, then

$$h = \frac{1}{2}gt^2 \quad \text{[u = 0 as the body starts from rest]} \quad \text{... (1)}$$

Now, as the distance covered in (t – 1) second is

$$h' = \frac{1}{2}g(t-1)^2 \quad \text{... (2)}$$

So from Equations (1) and (2) distance travelled in the last second.

$$h - h' = \frac{1}{2}gt^2 - \frac{1}{2}g(t-1)^2$$

i.e., $h - h' = \frac{1}{2}g(2t-1)$

But according to given problem as (h –h') $= \frac{h}{2}$

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i.e., $\left(\frac{1}{2}\right)h = \left(\frac{1}{2}\right)g(2t-1)$

or $\left(\frac{1}{2}\right)gt^2 = g(2t-1)$ [as from equation (1) $h = \left(\frac{1}{2}\right)gt^2$]

or $t^2 - 4t + 2 = 0$

or $t = [4 \pm \sqrt{(4^2 - 4 \times 2)}]/2$

or $t = 2 \pm \sqrt{2}$

or t = 0.59 s or 3.41 s

0.59 s is physically unacceptable as it gives the total time t taken by the body to reach ground lesser than one sec while according to the given problem time of motion must be greater than 1s.

so t = 3.41 s

and h = ½ × (9.8) × $(3.41)^2$ = 57 m

EXERCISE

44. If a body is thrown up with the velocity of 15 m/s then maximum height attained by the body is (g = 10 m/s^2)

(A) 11.25 m (B) 16.2 m

(C) 24.5 m (D) 7.62 m

45. A body falls from rest in the gravitational field of the earth. The distance travelled in the fifth second of its motion is $(g = 10 m/s^2)$

(A) 25m (B) 45m

(C) 90m (D) 125m

46. If a ball is thrown vertically upwards with speed u, the distance covered during the last t seconds of its ascent is

(A) $\frac{1}{2}gt^2$ (B) $ut - \frac{1}{2}gt^2$

(C) [u – gt]t (D) ut

47. A man throws balls with the same speed vertically upwards one after the other at an interval of 2 seconds. What should be the speed of the throw so that more than two balls are in the sky at any time (Given $g = 9.8 m/s^2$)

(A) At least 0.8 m/s (B) Any speed less than 19.6 m/s

(C) Only with speed 19.6 m/s (D) More than 19.6 m/s

48. A man drops a ball downside from the roof of a tower of height 400 meters. At the same time another ball is thrown upside with a velocity 50 meter/sec. from the surface of the tower, then they will meet at which height from the surface of the tower

(A) 100 meters (B) 320 meters

(C) 80 meters (D) 240 meters

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49. A very large number of balls are thrown vertically upwards in quick succession in such a way that the next ball is thrown when the previous one is at the maximum height. If the maximum height is 5m, the number of ball thrown per minute is (take $g = 10\,ms^{-2}$)

(A) 120 (B) 80

(C) 60 (D) 40

50. A particle is thrown vertically upwards. If its velocity at half of the maximum height is 10 m/s, then maximum height attained by it is (Take $g = 10$ m/s^2)

(A) 8 m (B) 10 m

(C) 12 m (D) 16 m

51. A stone is shot straight upward with a speed of 20 m/sec from a tower 200 m high. The speed with which it strikes the ground is approximately

(A) 60 m/sec (B) 65 m/sec

(C) 70 m/sec (D) 75 m/sec

52. A body freely falling from the rest has a velocity 'v' after it falls through a height 'h'. The distance it has to fall down for its velocity to become double, is

(A) $2h$ (B) $4h$

(C) $6h$ (D) $8h$

53. A body sliding on a smooth inclined plane requires 4 seconds to reach the bottom starting from rest at the top. How much time does it take to cover one-fourth distance starting from rest at the top

(A) 1 s (B) 2 s

(C) 4 s (D) 16 s

54. A stone dropped from a building of height h and it reaches after t seconds on earth. From the same building if two stones are thrown (one upwards and other downwards) with the same velocity u and they reach the earth surface after t_1 and t_2 seconds respectively, then

(A) $t = t_1 - t_2$ (B) $t = \frac{t_1 + t_2}{2}$

(C) $t = \sqrt{t_1 t_2}$ (D) $t = t_1^2 t_2^2$

55. By which velocity a ball be projected vertically downward so that the distance covered by it in 5th second is twice the distance it covers in its 6th second ($g = 10\,m/s^2$)

(A) 58.8 m/s (B) 49 m/s

(C) 65 m/s (D) 19.6 m/s

56. Water drops fall at regular intervals from a tap which is 5 m above the ground. The third drop is leaving the tap at the instant the first drop touches the ground. How far above the ground is the second drop at that instant

(A) 2.50 m (B) 3.75 m

(C) 4.00 m (D) 1.25 m

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57. A balloon is at a height of 81 m and is ascending upwards with a velocity of 12 m/s. A body of 2 kg weight is dropped from it. If g = 10 m/s^2, the body will reach the surface of the earth in

(A) 1.5 s (B) 4.025 s

(C) 5.4 s (D) 6.75 s

58. A particle is dropped under gravity from rest from a height h(g = 9.8 m/s^2) and it travels a distance 9h/25 in the last second, the height h is

(A) 100 m (B) 122.5 m

(C) 145 m (D) 167.5 m

59. A stone thrown upward with a speed u from the top of the tower reaches the ground with a velocity 3u. The height of the tower is

(A) $3u^2/5$ (B) $4u^2/g$

(C) $6u^2/g$ (D) $9u^2/g$

60. A stone dropped from the top of the tower touches the ground in 4 sec. The height of the tower is about

(A) 80 m (B) 40 m

(C) 20 m (D) 160 m

61. A body is released from a great height and falls freely towards the earth. Another body is released from the same height exactly one second later. The separation between the two bodies, two seconds after the release of the second body is

(A) 4.9 m (B) 9.8 m

(C) 19.6 m (D) 24.5 m

MOTION WITH VARIABLE ACCELERATION

(i) If acceleration is a function of time

$a = f(t)$ then $v = u + \int_0^t f(t)dt$ and $s = ut + \int\left(\int f(t)dt\right)dt$

(iii) If acceleration is a function of velocity

$a = f(v)$ then $t = \int_0^v \frac{dv}{f(v)}$ and $x = x_0 + \int_0^v \frac{vdx}{f(v)}$

SOLVED EXAMPLE

Example 41. An electron starting from rest has a velocity that increases linearly with the time that is v = kt, where k = 2 m/sec^2. The distance travelled in the first 3 seconds will be

(A) 9 m (B) 16 m

(C) 27 m (D) 36 m

Solution. **(A)**

$$x = \int_{t_1}^{t_2} vdt = \int_0^3 2tdt = 2\left[\frac{t^2}{2}\right]_0^3 = 9m.$$

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